



EE 232 Lightwave Devices Lecture 20: Noises in Photodetectors

Reading: Yariv 10.3-10.5 or Chuang 15.1.3

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Poisson Distribution

Poisson distribution:
a given event occurring in any time interval is distributed uniformly over the interval.

The probability of n electrons arriving in a period T : is

$$p(n) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$$

where \bar{n} is the average number of electrons arriving in T

Properties of Poisson Distribution:

$$\text{Mean} = \bar{n}$$

$$\text{Variance} = \bar{n}$$

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Spectral Density Function

Random variable $i(t)$ consists of a large number of individual events (e.g., single-electron photocurrent) at random time:

$$i(t) = \sum_{i=1}^{N_T} f(t-t_i), \quad 0 \leq t \leq T$$

$$\text{Fourier transform: } I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega)$$

$$F_i(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t-t_i) e^{-i\omega t} dt = \frac{e^{-i\omega t_i}}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = e^{-i\omega t_i} F(\omega)$$

$$\langle |I_T(\omega)|^2 \rangle = \langle |F(\omega)|^2 \left\{ N_T + \sum_{i=1}^{N_T} \sum_{j=1}^{N_T} e^{-i\omega(t_i-t_j)} \right\} \rangle = \overline{N_T} |F(\omega)|^2 = \overline{N} T |F(\omega)|^2$$

\overline{N} : average rate of electron arrival

$$\text{Spectral density function: } S(\nu) = \lim_{T \rightarrow \infty} \frac{8\pi^2 |I_T(2\pi\nu)|^2}{T} = 8\pi^2 \overline{N} |F(2\pi\nu)|^2$$

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Shot Noise

Shot Noise: Noise current arising from random generation and flow of mobile charge carriers.

Current pulse due to a single electron moving at $v(t)$:

$$i_e(t) = \frac{ev(t)}{d}$$

$$\text{Fourier transform: } F(\omega) = \frac{1}{2\pi} \frac{e}{d} \int_0^{t_a} v(t) e^{-i\omega t} dt$$

$$t_a: \text{arrival time, } x(0) = 0, \quad x(t_a) = d,$$

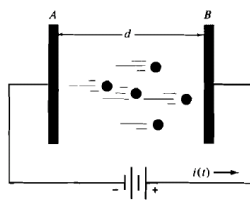
$$\text{Small transit time } t_a, \omega t_a \ll 1 \rightarrow e^{-i\omega t} \sim 1$$

$$F(\omega) = \frac{1}{2\pi} \frac{e}{d} \int_0^{t_a} \frac{dx}{dt} \cdot 1 \cdot dt = \frac{1}{2\pi} \frac{e}{d} \int_0^d dx = \frac{e}{2\pi}$$

$$S(\nu) = 8\pi^2 \overline{N} \left(\frac{e}{2\pi} \right)^2 = 2e\overline{I}$$

$$\overline{I} = e\overline{N}$$

$$\overline{i_N^2}(\nu) = S(\nu) d\nu = 2e\overline{I} d\nu$$

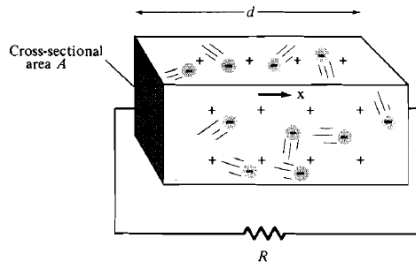


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Thermal Noise (Johnson Noise)



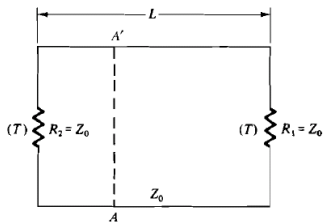
- Fluctuation in the voltage across a dissipative circuit element (resistor)
- Caused by thermal motion of charged carriers

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Thermal Noise Derivation



Consider two resistors connected by a lossless transmission line of length L :
 voltage wave: $v(t) = A \cos(\omega t \pm kz)$
 Assume periodic condition: $kL = 2m\pi$

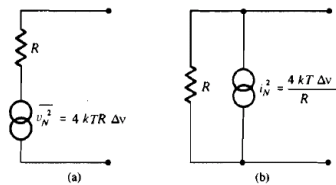
Mode density: $\rho(\nu) = \frac{L}{c}$

Power flow: $P = \frac{\text{Energy}}{\text{Transit Time}}$

$$P = \frac{1}{L/c} \left(\frac{L}{c} \Delta\nu \right) \left(\frac{h\nu}{e^{h\nu/k_B T} - 1} \right) = \frac{h\nu \Delta\nu}{e^{h\nu/k_B T} - 1}$$

$h\nu / k_B T \ll 1$

$$P = k_B T \Delta\nu = \left(\overline{v_N^2} \left(\frac{R}{R+R} \right)^2 \right) \frac{1}{R} = \left(\overline{i_N^2} \left(\frac{R}{R+R} \right)^2 \right) R$$



Equivalent mean square noise voltage: $\overline{v_N^2} = 4k_B T R \Delta\nu$

Equivalent mean square noise current: $\overline{i_N^2} = \frac{4k_B T \Delta\nu}{R}$

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Noise in p-i-n Photodiode

Noises in p-i-n photodiodes: shot noise and thermal noise

$$\overline{i_N^2(v)} = \overline{i_{N,shot}^2(v)} + \overline{i_{N,thermal}^2(v)} = 2e\bar{I}\Delta v + \frac{4k_B T \Delta v}{R}$$

Signal:

$$i_S^2(v) = \bar{I}^2$$

Signal to noise ratio (SNR):

$$SNR = \frac{\bar{I}^2}{2e\bar{I}\Delta v + \frac{4k_B T \Delta v}{R}}$$

Note that the SNR improves with increasing average photocurrent \bar{I}

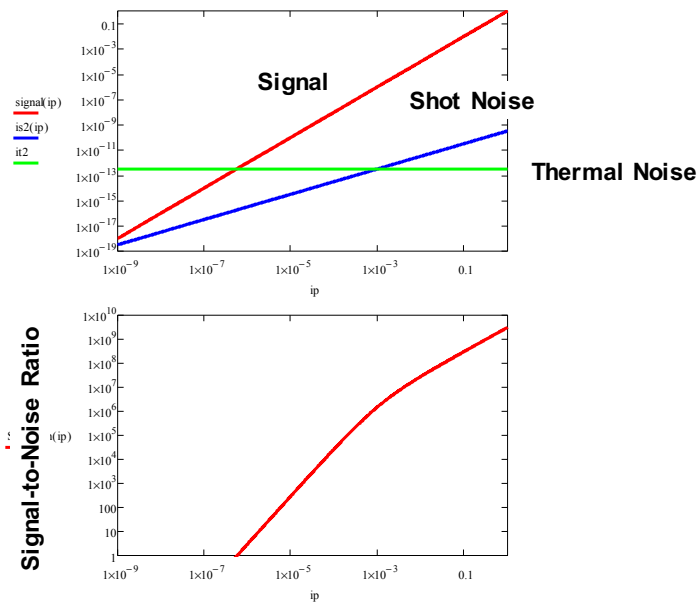
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Example

B = 1 GHz
R = 50



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